Intertemporal Choice

Enzo's intertemporal preferences can be described by the following instantaneous utility function $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and the rate of impatience is p. He must choose how to distribute his consumption between two periods knowing that the interest rate is r and that he only receives M_1 in income in the first period.

a) Find Enzo's Euler equation.

b) Show that if $p > r$ Enzo will consume more in the first period.

c) Show that even if $p > r$ and despite $M_2 = 0$ Enzo will never consume 0 in the second period.

d) Find the Marshallian demands for consumption in both periods if $\gamma > 0$. (Advice, do not minimize utility).

d) Assume that $\sigma = 0$ and solve again.

1. We calculate the derivatives:
$$u'_{c_1} = c_1^{-\sigma}$$
$$u'_{c_2} = c_2^{-\sigma}$$
The constraint is:
$$M_1 = c_1 + \frac{c_2}{1+r}$$
The Euler equation:
$$\frac{c_1^{-\sigma}}{c_2^{-\sigma}} = \frac{1+r}{1+\rho}$$
$$\frac{c_2^{\sigma}}{c_1^{\sigma}} = \frac{1+r}{1+\rho}$$
2. If $\rho > r$ then what happens is:
$$c_2^{\rho} < c_1^{\rho}$$
And therefore:
$$c_2 < c_1$$
3. For $c_2 = 0$ to happen, ρ would have to tend to infinity, which would mean that the individual is extremely impatient.
4. We clear from the Euler equation:
$$c_2 = c_1[\frac{1+r}{1+\rho}]^{1/\sigma}$$
We insert into the constraint:
$$M_1 = c_1 + \frac{1}{1+r}c_1[\frac{1+r}{1+\rho}]^{1/\sigma}$$
$$M_1 = c_1[1 + \frac{(1+r)^{1/\sigma-1}}{(1+\rho)^{1/\sigma}}]$$
$$M_1 = c_1[\frac{(1+\rho)^{1/\sigma} + (1+r)^{1/\sigma-1}}{(1+\rho)^{1/\sigma}}]$$
$$c_1 = \frac{M_1(1+\rho)^{1/\sigma}}{(1+\rho)^{1/\sigma} + (1+r)^{1/\sigma-1}}$$
$$c_2 = \frac{M_1(1+\rho)^{1/\sigma}}{(1+\rho)^{1/\sigma} + (1+r)^{1/\sigma-1}}[\frac{1+r}{1+\rho}]^{1/\sigma}$$
5. If $\sigma = 0$ then the utility function changes
$$u = c_1 + \frac{1}{1+\rho}c_2$$
It is a utility function that represents
6. If $\sigma = 0$ then the utility function changes to
$$u = c_1 + \frac{1}{1+\rho}c_2$$
It is a utility function that represents preferences for perfect substitutes, so the choice will be determined by comparing the coefficient ratio and the ratio of interest rates and rate of impatience:
$$\frac{1}{\frac{1}{1+\rho}}$$
$$\frac{1}{\frac{1}{1+r}}$$
Therefore, if $\rho > r$ the individual will consume only in period 1. If $r > \rho$ the individual will only consume in period 2.